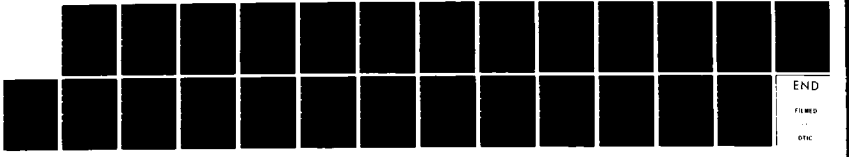
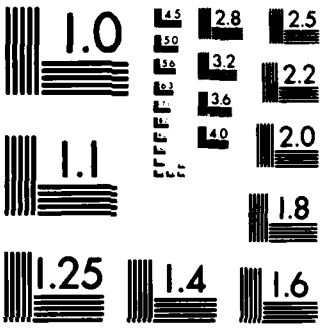
AD-A144 487 ZEEMAN STUDIES OF SHALLOW DONORS AND EXCITONS IN QUANTUM WELLS(U) NEW ORLEANS UNIV LA R L GREENE MAR 84 AFOSR-TR-84-0627 AFOSR-83-0120 1/1

UNCLASSIFIED F/G 20/10 NL





UNCLASSIFIED
SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: AFOSR-TR- 84-0627	2. GOVT ACCESSION NO.
3. RECIPIENT'S CATALOG NUMBER	(13)
4. TITLE (and Subtitle): Zeeman Studies of Shallow Donors and Excitons in Quantum Wells	5. TYPE OF REPORT & PERIOD COVERED: 01 MAY 83 - 29 FEB 84 FINAL
	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s): Dr. Ronald L. Greene, Dr. K.K. Bajaj	8. CONTRACT OR GRANT NUMBER(s): AFOSR-83-0120
9. PERFORMING ORGANIZATION NAME AND ADDRESS: University of New Orleans, New Orleans, LA 70148	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: 2306/D9 61102F
11. CONTROLLING OFFICE NAME AND ADDRESS: AFOSR/NE, Bolling AFB, DC 20332	12. REPORT DATE: Mar 84
	13. NUMBER OF PAGES: pp 1-22.
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)	15. SECURITY CLASS. (of this report): UNCLASSIFIED
	15a. DECLASSIFICATION DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Unlimited

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

See reverse

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

20. (cont.)

Abstract: A theoretical study has been made of the shallow donor and Wannier exciton within a one-dimensional quantum well. The variational method was used with a cylindrical Gaussian basis set. In order to facilitate comparison with future experimental measurements of excited states of these systems, an external magnetic field was assumed perpendicular to the interfaces between the barrier material and the well. Calculations reveal that the choice of matching conditions used at the interfaces has little effect on the binding energies of the ground or first few excited states of the shallow donor, except for well widths considerably smaller than the effective Bohr radius. The results of calculations of the shallow donor ground and first few excited states are presented for a variety of well sizes and magnetic field strengths. Similar results are given for the ground state of the Wannier exciton.

A-1

Zeeman Studies of Shallow Donors and Excitons in Quantum Wells

Final Report

Abstract: A theoretical study has been made of the shallow donor and Wannier exciton within a one-dimensional quantum well. The variational method was used with a cylindrical Gaussian basis set. In order to facilitate comparison with future experimental measurements of excited states of these systems, an external magnetic field was assumed perpendicular to the interfaces between the barrier material and the well. Calculations reveal that the choice of matching conditions used at the interfaces has little effect on the binding energies of the ground or first few excited states of the shallow donor, except for well widths considerably smaller than the effective Bohr radius. The results of calculations of the shallow donor ground and first few excited states are presented for a variety of well sizes and magnetic field strengths. Similar results are given for the ground state of the Wannier exciton.

I. Introduction

Recent advances in epitaxial crystal growth techniques have made it possible to grow systems consisting of alternate layers of two different semiconductors, with sharp interfaces. One of the most common of these layered systems consists of GaAs alternated with $Ga_{1-x}Al_xAs$. The difference in band gap of the two semiconductors leads to conduction and valence band discontinuities at the interfaces. This effectively produces a periodic square-well potential with period controllable during growth. This large scale periodic potential results in the breaking the usual electron energy bands into subbands, and has significant effects on the transport properties and bound, atomic-like systems of these materials. The research reported herein deals with the effects of quantum wells upon hydrogen-like systems, specifically shallow donors and Wannier excitons.

The study of such systems is still in its infancy. The first calculations of the ground state binding energy of shallow impurities in quantum wells assumed an infinite barrier between the well material and the barrier material.[1,2] Later calculations by Mailhiot et al.[3,4] and by Greene and Bajaj[5] replaced the infinite barriers by finite band discontinuities dependent upon x, the fractional Al content, and determined energies for the first few excited states as well as the ground state. The latter authors in particular found significant qualitative and quantitative differences between the behavior of 2s- and $2p_{\pm}$-like excited states compared to the $2p_o$-like state.

Assuming infinite quantum wells, Bastard et al.[6] performed similar calculations of the binding energy of the ground state of Wannier excitons in quantum well structures. Shortly afterward, Greene and Bajaj[7] reported results of the exciton ground state in finite-well GaAs-$Ga_{1-x}Al_xAs$ structures, and very recently extended the work to include a few low-lying excited states.[8]

One of the major reasons for calculating the binding energies of these hydrogen-like systems is the expectation that comparison between theory and experiment can give more information about the general quality of the layered materials and about the nature of the interface between the two semiconductors. However, it is difficult to experimentally determine donor and exciton excited state energies due to the large line widths of ground-to-excited-state transitions. In bulk GaAs these line widths can be reduced by the application of an external magnetic field.[9] It was the anticipation that a similar technique would be used in these quantum well structures that led to the investigation of the Zeeman levels of shallow donors and Wannier exciton in quantum wells.

The objectives of the research were the following:

1) To study how the treatment of the different semiconductor effective masses and dielectric constants affects the binding energies of shallow donor states,
2) To investigate the possibility of using the energy difference between the shallow donor $2p_{\pm}$-like and $2p_{o}$-like states to provide an independent determination of the conduction band discontinuity,
3) To determine the binding energies of the ground and several excited states of the shallow donor at the center of the GaAs-$Ga_{1-x}Al_xAs$ quantum well, as a function of well size and applied magnetic field,
4) To obtain results for the Wannier exciton ground state energy in the GaAs-$Ga_{1-x}Al_xAs$ quantum well as a function of well size and applied magnetic field.

The results are given in Section III, and specific accomplishments are summarized in Section IV of this report.

II. Theory and Method Used

A. Shallow Donor

The effective mass approximation is assumed valid for the GaAs-$Ga_{1-x}Al_xAs$ quantum wells under consideration. With this approximation the Hamiltonian for a hydrogenic donor in a magnetic field, applied in a direction prependicular to the layers, can be written

$$H=\frac{1}{2m^*}\left(-i\hbar\vec{\nabla}-\frac{e\vec{A}}{c}\right)^2-\frac{e^2}{\varepsilon_o r}+V'w(z)\ , \qquad (1)$$

The donor impurity is taken to be located at the center of the well. The well is a finite square well

$$V'_w(z)=\begin{cases}0 & |z| < L/2\\ V_o & |z| > L/2\ ,\end{cases} \qquad (2)$$

where L is the width of the quantum well, and V_o is the conduction band discontinuity. Using the cylindrical gauge, where

$$\vec{A}=\frac{1}{2}\vec{B}\times\vec{r},$$

and a cylindrical coordinate system with z-axis along the magnetic field direction, yields

$$H=-\frac{1}{m_e}\nabla^2-\frac{2}{r}+\gamma L_z+\frac{1}{4}\gamma^2\rho^2+V_w(z)\ , \qquad (3)$$

where

$$r=(\rho^2+z^2)^{\frac{1}{2}}$$

and L_z is the z-component of the angular momentum operator (in units of $\hbar$). The Hamiltonian in Eq. (3) is written in units of the effective Rydberg in bulk GaAs,

$$R^*=\frac{m^*e^4}{2\varepsilon_o^2\hbar^2},$$

and the unit of length is the GaAs effective Bohr radius,

$$a^*=\frac{\hbar^2\varepsilon_o}{m^*e^2}$$

The m_e in Eq. (3) is the effective mass of the electron in units of the bulk GaAs electron effective mass, m*. For the GaAs-$Ga_{1-x}Al_xAs$ structures, m_e=1 within the well, and m_e>1 (depending on x) in the barrier material. Finally,

the parameter γ is a dimensionless measure of the magnetic field, given by

$$\gamma = \frac{ehB}{2m^*cR^*}$$

The conduction band discontinuity (or barrier height)in Eq. (2) is assumed to be 85% of the total band gap discontinuity determined from the empirical expression [10]

$$\Delta E_g = 1.155x + .37x^2 \text{ eV}. \qquad (4)$$

For typical values of x and L, the barrier height is considerably larger than the effective Rydberg. Thus, the energy associated with the Coulomb interaction will, except for large well widths, be small compared to the subband energy due to the square well. Because of this, it is helpful to explicitly factor the solution to the ground state of an electron in the one-dimensional square well out of the donor wave function Ψ:

$$\Psi = f(z)G(\rho,z,\phi) \ . \qquad (5)$$

The (unnormalized) square well solution is given by

$$f(z) = \begin{cases} \cos kz & |z| < L/2 \\ Ae^{-\kappa|z|} & |z| > L/2 \end{cases} \qquad (6)$$

The parameter κ is determined from the energy of the first subband, and the constants A and κ are fixed by the matching conditions at the interface. It is assumed that F(z) and $(1/m)\,\partial f/\partial z$ are continuous across the interface. As discussed in SectionIII, however, the results are not very sensitive to the matching conditions chosen.

For wells less than several effective Bohr radii in width, the factorization in Eq.(5) is particularly appropriate because it enables the removal of the relatively large square well energy that otherwise would numerically overwhelm the smaller Coulomb energy contribution. For large well widths, where the energy of the subband is less than or of the same order as the Coulomb contribution, the factorization is of little or no value because higher subbands can be mixed with the lowest subband by the Coulomb potential. However, this does not cause any significant problem provided the function $G(\rho,z,\phi)$

has sufficient variational flexibility.

Since the Hamiltonian of Eq.(3) is cylindrically symmetric, the Z-component of the angular momentum is a good quantum number. The ϕ dependence of the wave function thus has the form $\exp(im\phi)$, where m is an integer. If the donor impurity is located at the center of the well, the Hamiltonian is also invariant under reflection through the origin. The wave function has a definite parity. Using this knowledge, the function $G(\rho,z,\phi)$ can be written in the form

$$G(\rho,z,\phi) = \rho^{|m|} e^{im\phi} z^q \sum_{i,j} A_{ij} G_{ij}(\rho,z) \quad . \tag{7}$$

The parity of this function is determined by m+q, where q = 0, 1.

The basis functions $G_{ij}(\rho, z)$ are taken to be products of Gaussians in ρ and z variables:

$$G_{ij}(\rho,z) = e^{-(\beta+\alpha_i)\rho^2} e^{-\alpha_j z^2} \quad . \tag{8}$$

This choice was made because of the success of a similar basis set which Aldrich and Greene[11] applied to the problem of hydrogen in a uniform magnetic field. They found that the Gaussian basis set yielded good results for the ground and several excited states throughout the range $0 \leq \gamma \leq 10$.

The set of parameters $\{\alpha_i\}$ used in this work is given in Table 1. They are taken from the results of Huzinaga,[12] who did a detailed study of the use of Gaussian basis functions in the calculation of electron energies in hydrogen. This set of $\{\alpha_i\}$ gives energies for the 1s, 2s and 2p free-hydrogen states accurate to within 0.001 Rydbergs. The parameter β was varied in each case to minimize the energy. It is primarily determined by the size of the magnetic field. Calculations were also made using β as a multiplicative variational parameter in the argument of the ρ Gaussian, rather than the additive one of Eq.(8). The results were poorer (gave smaller binding energies) by up to ten per cent, particularly for larger magnetic field strengths. The additive β differentially distorts the individual ρ Gaussians, in a way consistent with the physics of the magnetic field (which affects the bound electron least strongly when it is near the donor). The multiplicative β , by contrast,

provides the same scaling for each of the ρ Gaussians.

The results presented in the next section used $A_{ij}=0$ for $i\neq j$; the energies were then determined by solving the matrix eigenvalue equation,

$$\underset{\sim}{H}\underset{\sim}{\psi} = E\underset{\sim}{U}\underset{\sim}{\psi} ,$$

where $\underset{\sim}{H}$ and $\underset{\sim}{U}$ are the Hamiltonian and overlap matrices. For m=q=0 (even parity), the number of basis functions (and hence the order of the Hamiltonian and overlap matrices) is five. For m=1, q=0 and m=0, q=1 (odd parity), there are four basis functions.

A number of runs at various values of γ and L were also made with $A_{ij}=0$ for $|i-j|>1$ and non-zero otherwise. The energies were obtained as before. In this case there are 13 and 10 basis functions for the even and odd parity cases, respectively. The larger number of basis functions yielded improvements in the calculated binding energies of about 0.001 effective Rydbergs or less, which gives confidence in the accuracy of the results.

B. Wannier Exciton

If the quantum well associated with the valence band discontinuity is assumed large enough to split the degeneracy of the GaAs valence band, two excitons (known as the light-hole and heavy-hole excitons) arise. In each case the reduction of the Hamiltonian follows pretty much the same steps as for the donor. The major difference arises because the hole moves in the Coulomb potential and its own quantum well with an effective mass which is not isotropic. The quantity $m_{\pm}$ is taken to be the heavy (+) or light (-) hole effective mass in the direction perpendicular to the layers (Z-axis), and $\mu_{\pm}$ is the electron-hole reduced effective mass corresponding to the heavy (+) or light (-) hole bands in the plane perpendicular to the Z-axis. These masses are given in terms of the Kohn-Luttinger[13] band parameters γ_1 and γ_2 as[6]

$$\frac{1}{\mu_{\pm}} = \frac{1}{m_e} + (\gamma_1 \pm \gamma_2) \qquad (9a)$$

and

$$\frac{1}{m_{\pm}} = \gamma_1 \mp 2\gamma_2 . \qquad (9b)$$

The masses in these equations are in units of the free electron mass.

In terms of these masses, the Hamiltonian for the exciton in the quantum wells formed by the conduction and valence band discontinuities is given by

$$H = -\left[\frac{1}{\rho} \frac{\partial}{\partial \rho} \rho \frac{\partial}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2}{\partial \phi^2} \right] - \frac{\mu_\pm}{m_e} \frac{\partial^2}{\partial z_e^2} - \frac{\mu_\pm}{m_\pm} \frac{\partial^2}{\partial z_h^2} - \frac{2}{r} + \gamma L_z + \frac{1}{4}\gamma^2 \rho^2 + V_{ew}(z_e) + V_{hw}(z_h) . \quad (10)$$

The symbols ρ, ϕ, and r are electron-hole relative coordinates, and the Hamiltonian is expressed in units of the effective Rydberg defined with electron-hole reduced mass, $\mu_\pm$.

Since the exciton Hamiltonian of Eq. (10) has cylindrical symmetry (as does that for the donor, Eq. (3)) the same type of variational solution was chosen. The only difference is that the ground state solution for the hole in a one-dimensional square well was factored out of the variational trial function, in addition to that for the electron:

$$\Psi(\vec{r_e}, \vec{r_h}) = f_e(z_e) f_h(z_h)\, G(\rho, z, \phi). \quad (11)$$

The function $G(\rho, z, \phi)$, which is assumed to be a function only of electron-hole relative coordinates, is of the same form as Eq. (7) and (8). For the results of the next section $A_{ij} = 0$ for $i \neq j$, and the A_{ii} are determined from the solution of the eigenvalue problem.

The set of parameters $\{\alpha_i\}$ is the same as before since the calculation is done using effective atomic units. However, the Hamiltonian and overlap matrices now involve integrals over both electron and hole positions.

III. Results and discussion

To examine the importance of the different effective masses of GaAs and $Ga_{1-x}Al_xAs$, two separate sets of calculations of the donor binding energy of the ground and several excited states were performed for x=.15, .30; γ =0.0,

.1, .2, .3, .5, .75, 1.0, 2.0, 5.0; and various values of L from 50-1000Å. One set of calculations used the GaAs value for the electron effective mass in both the barrier and well semiconductors. The interface matching conditions then required continuity of the function f of Eq. (6) and $\partial f/\partial z$ across the interface. These are the conditions assumed by Greene and Bajaj.[5] The other set of calculations used the following expression for the $Ga_{1-x}Al_xAs$ effective mass (m_e, in units of the free electron mass):[4]

$$m_e = 0.067 + 0.83x \qquad (11)$$

The matching conditions at the interface in this case were that $f(z)$ and $(1/m_e)\,\partial f/\partial z$ are continuous across the interface. These conditions are similar to those used by Mailhiot et al.[3,4] and have been justified for the GaAs-$Ga_{1-x}Al_xAs$ system by the band calculations of Ando and Mori.[14]

The differences between the binding energies obtained in the two sets of calculations are very small. In most cases they differ by less than 1%. The worst case occurs for x=.15, L=50Å with small γ. The differences there are about 3% for the 1s-like and $2p_{\pm}$-like states. Thus it appears that as far as the binding energy of the shallow donor is concerned, it does not matter much what matching conditions are used, except for very small L.

The original proposal for this work indicated that a study of the effects of the different dielectric constants of the two semiconductors would be undertaken. This intention was prompted by the incorrect treatment of the potential with the two dielectric constants in the early work of Mailhiot et al.[4] After the proposal was submitted, Maihiot et al. published the results of more complete calculations[5] which corrected the error. Because their results are very close to the results of Greene and Bajaj[5] who used the same dielectric constant for both GaAs and $Ga_{1-x}Al_xAs$, it was decided that additional calculations using different dielectric constants would not be worthwhile.

With regard to the second research objective, a number of calculations of the binding energy of the $2p_{\pm}$-like and $2p_o$-like states were made, varying the barrier height. Two different well widths, L=500Å and L=750Å were used. The conduction band discontinuity was varied in 5% increments from 70% to 90% of the total band gap discontinuity, ΔE_g. The hope was that because of the significantly different behavior of these two states as a function of L, they could be used in conjunction with experiment to accurately determine the conduction band discontinuity. The results of these calculations indicate that this is likely not the case. The energy difference between the two states varies rather slowly with barrier height; the change form $.7\Delta E_g$ to $.9\Delta E_g$ is only 0.01 R*, or 0.058 meV. This small variation is not enough larger than the expected accuracy of the calculations to be acceptable.

Tables 2 - 7 give the results of calculations of the binding energies of the 1s-like, $2p_{\pm}$-like, and $2p_o$-like shallow donor states for a variety of well widths (L) and magnetic field strengths (γ). Two values of Al fractions, x=.15 and x=.30, are tabulated. Tables 8-11 give similar results for the ground state binding energies of the light-and heavy-hole exciton. Accuracy of all results is estimated to be about 0.01 effective Rydbergs.

IV. Summary

The major accomplishments of the research are listed below:

1. The binding energy of the donor ground and first few excited states has been shown to be very insensitive to the interface matching conditions used to take into account the affective mass difference between electrons in GaAs and those in $Ga_{1-x}Al_xAs$, at least for well widths between 50 and 1000 Å.
2. The energy difference between the $2p\pm$-like state and the $2p_o$-like state is too weakly dependent on the conduction band discontinuity to use that difference as a diagnostic.
3. The binding energies of the shallow donor ground and first few excited states have been tabulated for a variety of well widths and heights and magnetic field strengths.
4. The ground state binding energies of a Wannier exciton in a quantum well has similarly been tabulated for various well widths and heights and magnetic field strengths.

The work was performed by the principal investigator, Dr. Ronald L. Greene in collaboration with Dr. K. K. Bajaj of the Air Force Avionics Laboratory. Two manuscripts reporting the results of these investigations are currently be written and will be submitted for publication in Physical Review B.

REFERENCES

1. G. Bastard, Phys. Rev. B., 24, 4714 (1981).
2. R. C. Miller, D. A. Kleinman, W. T. Tsang, and A. C. Gossard, Phys. Rev. B , 24, 1134 (1981).
3. C. Mailhiot, Yia-Chung Chang, and T. C. McGill, Surf. Sci., 113, 161 (1982).
4. C. Mailhiot, Yia-Chung Chang, and T. C. McGill, Phys. Rev. B, 26, 4449 (1982).
5. Ronald L. Greene and K. K. Bajaj, Solid State Commun., 45, 825 (1983).
6. G. Bastard, E. E. Mendez, L. L. Chang, and L. Esaki, Phys. Rev. B, 26, 1974 (1982).
7. Ronald L. Greene and K. K. Bajaj, Solid State Commun., 45, 831 (1983).
8. Ronald L. Greene, Krishan K. Bajaj, and Dwight E. Phelps, Phys. Rev. B , in press.
9. David M. Larsen, Phys. Rev. B, 8, 535 (1973)
10. H. J. Lee, L. Y. Juravel, J. C. Woolley, and A. J. Springthorpe, Phys. Rev. B , 21, 659 (1980).
11. C. Aldrich and R. L. Greene, Phys. Stat. Sol. (b), 93, 343 (1979).
12. Sigeru Huzinaga, J. Chem. Phys., 42, 1293 (1965).
13. J. M. Luttinger and W. Kohn, Phys. Rev., 97, 869 (1955).
    J. M. Luttinger, Phys. Rev., 102, 1030 (1956).
14. T. Ando and S. Mori, Surf. Sci., 113, 124 (1982).

i	m=q=0 α_i	m+q=1 α_i
1	13.4	.734
2	2.01	.174
3	.454	.0557
4	.123	.0201
5	.0267	

Table 1. Values of α_i used for the ground state (m=q=0), and 2p-like states (m+q=1) in units of the inverse effective Bohr radius.

γ	0.0	0.1	0.2	0.3	0.5	0.75	1.0	2.0	5.0
L (Å)									
50	2.377	2.476	2.570	2.661	2.831	3.027	3.207	3.805	5.019
100	2.115	2.213	2.306	2.395	2.561	2.748	2.917	3.467	4.548
250	1.620	1.717	1.807	1.891	2.044	2.210	2.356	2.809	3.644
500	1.285	1.381	1.468	1.548	1.689	1.838	1.967	2.360	3.088
750	1.150	1.246	1.332	1.410	1.547	1.692	1.816	2.199	2.915
1000	1.089	1.184	1.270	1.347	1.484	1.628	1.751	2.133	2.848

Table 2. Binding energies of the ground state of the shallow donor in a GaAs-$Ga_{1-x}Al_xAs$ quantum well, with x=.15. Units are GaAs effective Rydbergs (5.81 meV). The parameter γ is proportional to the magnetic field (see text).

γ	0.0	0.1	0.2	0.3	0.5	0.75	1.0	2.0	5.0
L(Å)									
50	.446	.621	.757	.870	1.053	1.250	1.413	1.930	2.942
100	.431	.602	.736	.846	1.025	1.206	1.358	1.822	2.700
250	.404	.573	.700	.802	.967	1.133	1.268	1.658	2.339
500	.366	.530	.647	.739	.883	1.022	1.134	1.458	1.992
750	.337	.496	.606	.691	.823	.948	1.050	1.346	1.869
1000	.314	.471	.576	.657	.782	.902	1.000	1.287	1.805

Table 3. Binding energies of the $2p_{\pm}$-like state of the shallow donor in a GaAs-$Ga_{1-x}Al_xAs$ quantum well, with x=.15. Units are GaAs effective Rydberge (5.81 meV). The parameter γ is proportional to the magnetic field (see text).

γ	0.0	0.1	0.2	0.3	0.5	0.75	1.0	2.0	5.0
L (Å)									
400								.063	.315
500					.076	.160	.223	.380	.579
600		.061	.129	.184	.266	.339	.394	.528	.693
750	.130	.216	.280	.328	.400	.462	.508	.618	.754
1000	.231	.314	.371	.413	.473	.524	.561	.650	.762

Table 4. Binding energies of the $2p_o$-like state of the shallow donor in a GaAs-$Ga_{1-x}Al_xAs$ quantum well, with x=.15. Blank spaces indicate no bound state found. Units are GaAs effective Rydbergs (5.81 meV). The parameter γ is proportional to the magnetic field (see text).

γ	0.0	0.1	0.2	0.3	0.5	0.75	1.0	2.0	5.0
L (Å)									
50	2.560	2.658	2.762	2.845	3.026	3.228	3.403	3.984	5.270
100	2.188	2.292	2.386	2.476	2.643	2.833	3.006	3.567	4.630
250	1.650	1.746	1.837	1.922	2.075	2.243	2.389	2.849	3.696
500	1.295	1.391	1.479	1.559	1.700	1.849	1.978	2.373	3.103
750	1.155	1.250	1.336	1.415	1.552	1.696	1.821	2.204	2.920
1000	1.091	1.186	1.272	1.350	1.486	1.630	1.754	2.135	2.850

Table 5. Binding energies of the ground state of the shallow donor in a GaAs-$Ga_{1-x}Al_xAs$ quantum well, with x=.30. Units are GaAs effective Rydbergs (5.81 meV). The parameter γ is proportional to the magnetic field (see text).

γ	0.0	0.1	0.2	0.3	0.5	0.75	1.0	2.0	5.0
L (Å)									
50	.448	.623	.759	.873	1.062	1.256	1.420	1.940	2.975
100	.433	.606	.739	.849	1.029	1.211	1.364	1.836	2.729
250	.407	.576	.703	.806	.971	1.135	1.270	1.668	2.359
500	.368	.532	.649	.742	.887	1.027	1.140	1.465	2.021
750	.338	.498	.608	.693	.825	.951	1.054	1.350	1.872
1000	.316	.473	.578	.658	.784	.904	1.002	1.289	1.807

Table 6. Binding energies of the $2p_{\pm}$-like state of the shallow donor in a GaAs-$Ga_{1-x}Al_xAs$ quantum well, with x=.30. Units are GaAs effective Rydbergs (5.81 meV). The parameter γ is proportional to the magnetic field (see text).

γ	0.0	0.1	0.2	0.3	0.5	0.75	1.0	2.0	5.0
L (Å)									
400								.010	.268
500					.047	.131	.196	.356	.560
600		.043	.112	.167	.250	.325	.380	.516	.684
750	.112	.208	.272	.321	.394	.456	.502	.613	.750
1000	.223	.311	.368	.411	.471	.522	.559	.650	.763

Table 7. Binding energies of the $2p_o$-like state of the shallow donor in a GaAs-$Ga_{1-x}Al_xAs$ quantum well, with x=.30. Blank space indicate no bound state found. Units are GaAs effective Rydbergs (5.81 meV). The parameter γ is proportional to the magnetic field (see text).

γ	0.0	0.1	0.2	0.3	0.5	0.75	1.0	2.0	5.0
L (Å)									
50	2.388	2.517	2.614	2.707	2.880	3.079	3.261	3.869	5.106
100	2.187	2.323	2.419	2.510	2.680	2.872	3.047	3.622	4.765
250	1.732	1.860	1.953	2.039	2.197	2.370	2.524	3.013	3.932
500	1.420	1.527	1.616	1.698	1.845	2.005	2.143	2.573	3.383
750	1.291	1.398	1.477	1.557	1.701	1.855	1.989	2.406	3.198
1000	1.227	1.323	1.409	1.488	1.615	1.790	1.923	2.338	3.126

Table 8. Binding energy of the ground state of the heavy-hole exciton in a GaAs-$Ga_{1-x}Al_xAs$ quantum well, with x=.15. Units are GaAs effective Rydbergs (3.49 meV). The parameter γ is proportional to the magnetic field (see text).

γ	0.0	0.1	0.2	0.3	0.5	0.75	1.0	2.0	5.0
L (Å)									
50	1.713	1.898	1.987	2.070	2.224	2.398	2.553	3.079	4.044
100	1.721	1.918	2.011	2.096	2.252	2.427	2.583	3.102	4.058
250	1.416	1.602	1.696	1.780	1.930	2.092	2.231	2.670	3.460
500	1.158	1.311	1.395	1.474	1.613	1.759	1.883	2.263	2.945
750	1.046	1.172	1.258	1.336	1.472	1.612	1.730	2.091	2.752
1000	.990	1.107	1.193	1.269	1.401	1.539	1.656	2.011	2.667

Table 9. Binding energy of the ground state of the light-hole exciton in a GaAs-$Ga_{1-x}Al_xAs$ quantum well, with x=.15. Units are GaAs effective Rydbergs (4.40meV). The parameter γ is proportional to the magnetic field (see text).

γ	0.0	0.1	0.2	0.3	0.5	0.75	1.0	2.0	5.0
L (Å)									
50	2.568	2.690	2.788	2.881	3.059	3.263	3.451	4.086	5.403
100	2.269	2.404	2.500	2.592	2.763	2.959	3.137	3.726	4.908
250	1.758	1.887	1.981	2.067	2.225	2.400	2.555	3.049	3.981
500	1.430	1.537	1.627	1.709	1.856	2.016	2.154	2.586	3.397
750	1.296	1.394	1.482	1.563	1.706	1.861	1.995	2.413	3.205
1000	1.230	1.326	1.414	1.494	1.636	1.790	1.923	2.338	3.126

Table 10. Binding energy of the ground state of the heavy-hole exciton in a GaAs-$Ga_{1-x}Al_xAs$ quantum well, with x=.30. Units are GaAs effective Rydbergs (3.49 meV). The parameter γ is proportional to the magnetic field (see text).

γ	0.0	0.1	0.2	0.3	0.5	0.75	1.0	2.0	5.0
L (Å)									
50	1.992	2.190	2.283	2.370	2.534	2.721	2.892	3.458	4.552
100	1.867	2.071	2.166	2.254	2.416	2.600	2.766	3.306	4.333
250	1.457	1.646	1.741	1.826	1.978	2.142	2.284	2.732	3.541
500	1.170	1.327	1.411	1.490	1.630	1.776	1.901	2.282	2.967
750	1.052	1.178	1.265	13.43	1.478	1.618	1.737	2.098	2.757
1000	.992	1.110	1.196	1.272	1.404	1.541	1.658	2.014	2.667

Table 11. Binding energy of the ground state of the light-hole exciton in a GaAs-$Ga_{1-x}Al_xAs$ quantum well, with x=.30. Units are GaAs effective Rydbergs (4.40 meV). The parameter γ is proportional to the magneitc field (see text).